TABLE I
POSSIBLE TESTING EQUATIONS

Continuity of:	Testing Modes	Testing Cross Section	Describing Equation
$\mathbf{E}_{tangential}$	M modes of guide #2	S_2	Equation (6) of [1]
$\mathbf{E}_{tangential}$	M modes of guide #2	S_1	Equation (6) of [1], as the tangential electric field vanishes on $(S_2 - S_1)$
$\mathbf{E}_{tangential}$	N modes of guide #1	S_2	Irrelevant, as these modes are not defined on $(S_2 - S_1)$
$\mathbf{E}_{tangential}$	N modes of guide #1	S_1	Equation (7) of [1]
$\mathbf{H}_{tangential}$	M modes of guide #2	S_2	***The possibility we described above***
$\mathbf{H}_{tangential}$	M modes of guide #2	S_1	Equation (15) of [1]
$\mathbf{H}_{tangential}$	N modes of guide #1	S_2	Irrelevant, as these modes are not defined on $(S_2 - S_1)$
$\mathbf{H}_{tangential}$	N modes of guide #1	S_1	Equation (14) of [1]

the same spatial resolution everywhere, the cutoff frequencies of the highest order modes in either guides have to be more or less equal. This should apply to the short-circuited area $(S_2 - S_1)$ as well. In other words, if the surface magnetic current $\mathbf{J}_s = \mathbf{n} \times \mathbf{H}_t$ is to be expanded in terms of the eigenmodes of a virtual waveguide with the cross section $(S_2 - S_1)$, a number of modes $(M - N)$ should be used.

The standard mode-matching technique tests the continuity of the tangential electric field by the M modes of the larger guide and the continuity of the tangential magnetic field by the N modes of the smaller guide. This gives rise to $(N + M)$ linear equations relating the $(2N + 2M)$ modal expansion coefficients (N modal voltages and N modal currents in guide #1 in addition to M modal voltages and M modal currents in guide #2).

The possibility of testing both continuities by the M modes of the larger guide addressed in the comments of Solano *et al.* is, in fact, an extension of the standard mode-matching technique described above. It would result in $2M$ linear equations instead of the standard $(N + M)$ ones. The additional $(M - N)$ equations can, however, be used to determine $\mathbf{J}_s$, as $\mathbf{J}_s$ can be expanded in terms of the first $(M - N)$ modal magnetic fields of the virtual waveguide with the cross section $(S_2 - S_1)$, as mentioned above.

We do not, however, agree with the last sentence of the comments of Solano *et al.*, i.e., that testing the continuity of the tangential magnetic field by the M modes of the larger guide would result in a wrong formulation if the testing cross section is the smaller one (S_1) [(15) in the above paper[1]], as we have proven the correctness of this equation in the above paper. In order to clarify the whole situation, let us summarize all possible testing equations (a total number of eight possibilities), as shown in Table I.

[1]G. V. Eleftheriades, A. S. Omar, L. P. B. Katehi, and G. M. Rebeiz, *IEEE Trans. Microwave Theory Tech.*, vol. 42, no. 10, pp. 1896–1903, Oct. 1994.

Comments on "Relationship Between Group Delay and Stored Energy in Microwave Filters"

Christoph Ernst and Vasil Postoyalko

In the above paper,[1] it is shown that the time-averaged stored energy in a passive lossless reciprocal symmetrical or antimetrical two-port is proportional to the group delay. This is expressed in (29) in the above paper, i.e.,

$$W_{\text{av,tot}} = -|a_1|^2 \frac{d\phi_{21}}{d\omega}. \tag{1}$$

In a private communication, Cuthbert pointed out that "*[...] it appears that the same results you obtained were previously reported in the book by Paul Penfield, Robert Spence, and Simon Duinker* [1]. *The relevant pages are 64 to 67, especially Section 5.17 Group Delay and Stored Energy. It is interesting to note Penfield et al. attributing some particular results to Dicke* [2], *Kishi and Nakazawa* [3], *and Carlin* [4], *which you did not reference*." Reference [1, eq. (5.82)],

$$\frac{d\theta_{12}}{d\omega} + \frac{d\theta_{21}}{d\omega} = \frac{(W_e + W_m)_1 + (W_e + W_m)_2}{P} \tag{2}$$

which is attributed to Carlin [4], expresses that "*[...] the sum of the group-delays in the two directions, which may be regarded as the 'round-trip delay', is equal to the sum of the energies stored per watt input when the network is excited from each port.*" In the case when the two-port is reciprocal and symmetrical (or antimetrical), the relationship given in equation (29) in the above paper can be deduced directly from this equation. We were unaware of the work in [1]–[4] at the time and we would like to apologize for claiming credit for this result.

REFERENCES

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[4] H. J. Carlin, "Network theory without circuit elements," *Proc. IEEE*, vol. 55, pp. 482–497, Apr. 1967.

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[1]C. Ernst, V. Postoyalko, and N. G. Khan, *IEEE Trans. Microwave Theory Tech.*, vol. 49, no. 1, pp. 192–196, Jan. 2001.